UNIT---I-PART2

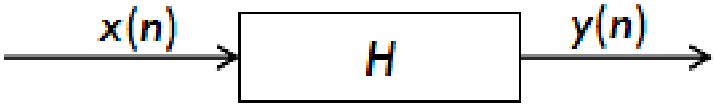
DR K.B.Khanchandani
Professor
SSGMCE SHEGAON

Overview

- Convolution
- Tabular Digital Convolution
- Boundary Effects
- Graphical Digital Convolution
- Convolution by Formula Method
- Properties of Convolution
- Correlation
- Cross Correlation
- Auto Correlation

Convolution

- Convolution combines an input x[n] with a system impulse response h[n] to produce a filter output y[n].



- This statement may be expressed as **y[n] = x[n] * h[n]** defined as:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

- or, equivalently,

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

This sum of products (or convolution sum) is in fact a function of n that represents the overlap between x[n] and the time-reversed and shifted version of h[n].

Convolution

The number of samples *N* in the output signal y[n] will be

$$N = M_1 + M_2 - 1$$

Where,
M_1 is the number of samples in sequence x[x]
M_2 is the number of samples in sequence h[x]

Difference Equation & Convolution

- The general form of the recursive difference equation is

$$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$

- The general form of the convolution is

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

- So the convolution has non-recursive relation with the difference equation.

Convolution

Digital convolution can be performed by the following methods

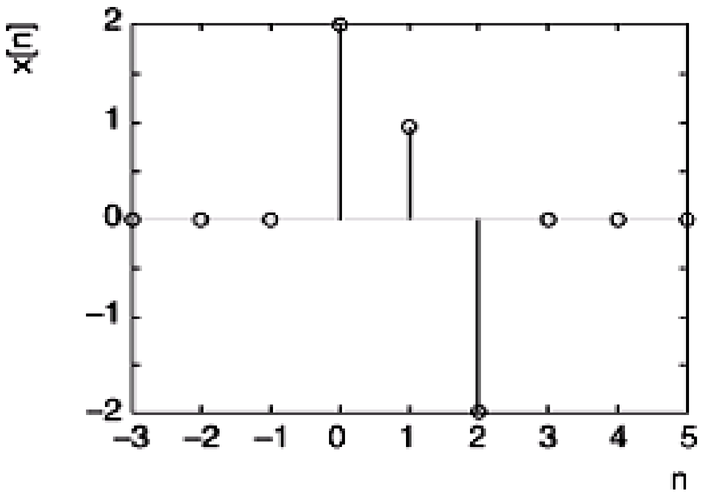
- Tabular method
- Graphical method
- Formula method

Tabular Digital Convolution

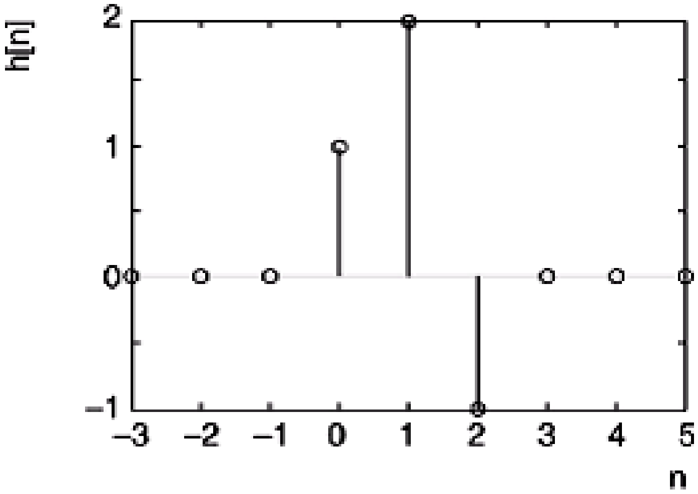
- **Step-1:** List the index $\boldsymbol{k}$ covering a sufficient range.
- **Step-2:** List the input $\boldsymbol{x[k]}$
- **Step-3:** Obtain the reversed sequence $\boldsymbol{h[-k]}$, and align the rightmost element of $\boldsymbol{h[n-k]}$ to the leftmost element of x[n].
- **Step-4:** Cross-multiply and sum the nonzero overlap terms to produce y[n].
- **Step-5:** Slide $\boldsymbol{h[n-k]}$ to the right by one position.
- **Step-6:** Repeat Step 4; stop if all the output values are zero or if required.

Tabular Digital Convolution

Example-1: Write the equation of following signals in the graphs.



(a) Input



(b) Impulse Response

Solution

a) $x[n] = 2\delta[n] + \delta[n-1] - 2\delta[n-2]$ **or**

$x[n] = [2, 1, 2]$

b) $h[n] = \delta[n] + 2\delta[n-1] - \delta[n-2]$ **or**

$h[n] = [1, 2, -1]$

Tabular Digital Convolution

Example-1: Find the output if x[n] = [2, 1, 2], and h[n] = [1, 2, -1].

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$x[n]$:			2	1	−2				
$h[-n]$:	−1	2	1						$y[0]=2$
$h[1-n]$:		−1	2	1					$y[1]=5$
$h[2-n]$:			−1	2	1				$y[2]=-2$
$h[3-n]$:				−1	2	1			$y[3]=-5$
$h[4-n]$:					−1	2	1		$y[4]=2$
$h[5-n]$:						−1	2	1	$y[5]=0$

The output is the sum of the products of the input samples and the impulse response samples.

Y[n] = [2, 5, -2, -5, 2]

Tabular Digital Convolution

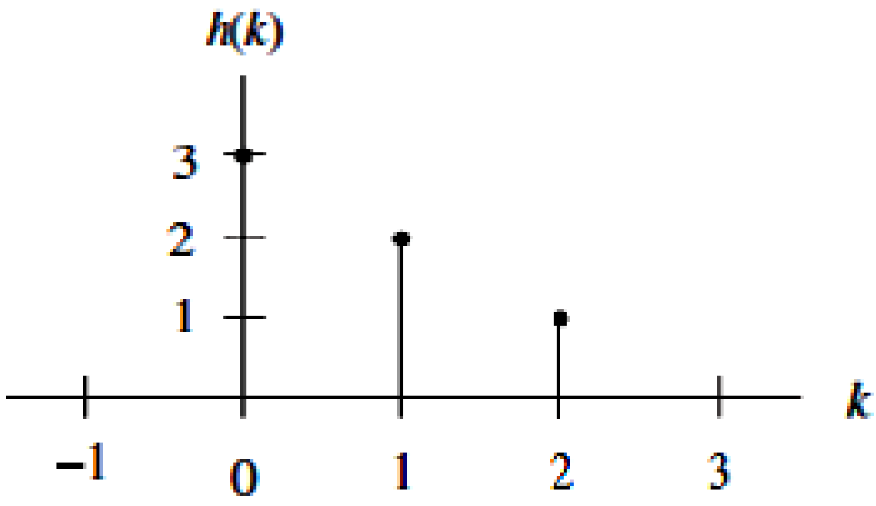
Example-2: Find the output using convolution if $x[n] = [1, 2, 3, 1]$, and $h[n] = [1, 2, 1, -1]$.

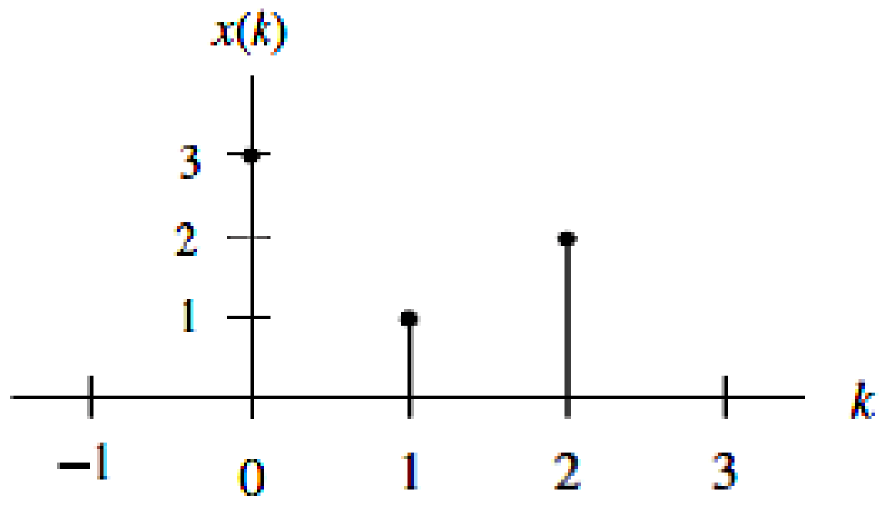
k	-3	-2	-1	0	1	2	3	4	5	6	
h(-1-k)	-1	1	2	1							y(-1)=1
h(-k)		-1	1	2	1						y(0)=4
x(k)				1	2	3	1				
h(1-k)			-1	1	2	1					y(1)=8
h(2-k)				-1	1	2	1				y(2)=8
h(3-k)					-1	1	2	1			y(3)=3
h(4-k)						-1	1	2	1		y(4)=-2
h(5-k)							-1	1	2	1	y(5)=-1

$$y(n) = x(n) * h(n) = \{1, \underset{\uparrow}{4}, 8, 8, 3, -2, -1\}$$

Tabular Digital Convolution

Example-3: Using the sequences defined in the following figure, evaluate the digital convolution by the tabular method.





$$x[k] = [\underset{\uparrow}{3}\ 1\ 2] \qquad h[k] = [\underset{\uparrow}{3}\ 2\ 1]$$

Tabular Digital Convolution

k:	-2	-1	0	1	2	3	4	5
x[k]:			3	1	2			
h[-k]:	1	2	3					
h[1-k]:		1	2	3				
h[2-k]:			1	2	3			
h[3-k]:				1	2	3		
h[4-k]:					1	2	3	
h[5-k]:						1	2	3

$y[0] = 3 \times 3 = 9$

Tabular Digital Convolution

k:	-2	-1	0	1	2	3	4	5
x[k]:			3	1	2			
h[-k]:	1	2	3					
h[1-k]:		1	2	3				
h[2-k]:			1	2	3			
h[3-k]:				1	2	3		
h[4-k]:					1	2	3	
h[5-k]:						1	2	3

$y[0] = 3 \times 3 = 9$

$\boxed{y[1] = 3 \times 2 + 1 \times 1 = 9}$

Tabular Digital Convolution

k:	-2	-1	0	1	2	3	4	5
x[k]:			3	1	2			
h[-k]:	1	2	3					
h[1-k]:		1	2	3				
h[2-k]:			1	2	3			
h[3-k]:				1	2	3		
h[4-k]:					1	2	3	
h[5-k]:						1	2	3

$y[0] = 3 \times 3 = 9$

$y[1] = 3 \times 2 + 1 \times 1 = 9$

$\boxed{y[2] = 3 \times 1 + 1 \times 2 + 2 \times 3 = 11}$

Tabular Digital Convolution

k:	-2	-1	0	1	2	3	4	5
x[k]:			3	1	2			
h[-k]:	1	2	3					
h[1-k]:		1	2	3				
h[2-k]:			1	2	3			
h[3-k]:				1	2	3		
h[4-k]:					1	2	3	
h[5-k]:						1	2	3

y[0] = 3 x 3 = 9

y[1] = 3 x 2 + 1 x 1 = 9

y[2] = 3 x 1 + 1 x 2 + 2 x 3= 11

$\boxed{y[3] = 1 \text{ x } 1 + 2 \text{ x } 2 = 5}$

Tabular Digital Convolution

k:	-2	-1	0	1	2	3	4	5
x[k]:			3	1	2			
h[-k]:	1	2	3					
h[1-k]:		1	2	3				
h[2-k]:			1	2	3			
h[3-k]:				1	2	3		
h[4-k]:					1	2	3	
h[5-k]:						1	2	3

$y[0] = 3 \times 3 = 9$

$y[1] = 3 \times 2 + 1 \times 1 = 9$

$y[2] = 3 \times 1 + 1 \times 2 + 2 \times 3 = 11$

$y[3] = 1 \times 1 + 2 \times 2 = 5$

$\boxed{y[4] = 2 \times 1 = 2}$

Tabular Digital Convolution

k:	-2	-1	0	1	2	3	4	5
x[k]:			3	1	2			
h[-k]:	1	2	3					
h[1-k]:		1	2	3				
h[2-k]:			1	2	3			
h[3-k]:				1	2	3		
h[4-k]:					1	2	3	
h[5-k]:						1	2	3

y[0] = 3 x 3 = 9

y[1] = 3 x 2 + 1 x 1 = 9

y[2] = 3 x 1 + 1 x 2 + 2 x 3= 11

y[3] = 1 x 1 + 2 x 2 = 5

y[4] = 2 x 1 = 2

y[5] = 0 (no overlap)

Y[n] = [9, 9, 11, 5, 2]

Tabular Digital Convolution

Example-4: Convolve the following two rectangular sequences using the tabular method.

$$x(n) = \begin{cases} 1 & n = 0, 1, 2 \\ 0 & otherwise \end{cases} \quad and \quad h(n) = \begin{cases} 0 & n = 0 \\ 1 & n = 1, 2 \\ 0 & otherwise \end{cases}$$

$$x(n) = \{\underset{\uparrow}{1}, 1, 1\}$$

$$h(n) = \{\underset{\uparrow}{0}, 1, 1\}$$

k :	**−2**	**−1**	**0**	**1**	**2**	**3**	**4**	**5**
$x(k)$:			1	1	1			
$h(-k)$:	1	1	0					
$h(1-k)$		1	1	0				
$h(2-k)$			1	1	0			
$h(3-k)$				1	1	0		
$h(4-k)$					1	1	0	
$h(n-k)$						1	1	0

$y(0) = 0$ (no overlap)

$y(1) = 1 \times 1 = 1$

$y(2) = 1 \times 1 + 1 \times 1 = 2$

$y(3) = 1 \times 1 + 1 \times 1 = 2$

$y(4) = 1 \times 1 = 1$

$y(n) = 0, n \geq 5$ (no overlap)

Stop

Tabular Digital Convolution

Exercise-1: Find the convolution of the two sequences x[n] and h[n] given by,

$$x[n] = [\underset{\uparrow}{1} \;\; 2 \;\; 4] \qquad h[n] = [\underset{\uparrow}{1} \;\; 1 \;\; 1 \;\; 1 \;\; 1]$$

Exercise-2: Find the convolution of the two sequences x[n] and h[n] given by,

$$x[\mathrm{n}] = \{\underset{\uparrow}{0} \;\; 1 \;\; -2 \;\; 3 \;\; -4\} \qquad h[n] = [0.5 \;\; 1 \;\; \underset{\uparrow}{2} \;\; 1 \;\; 0.5]$$

Tabular Digital Convolution

Exercise-3: determine the output for the first three samples of h[n] using the tabular method. Where x[n] = u[n] and h[n] = $(0.25)^n$u[n]

Solution

$$u(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1, 1, \ldots\} \qquad h(n) = \{\underset{\uparrow}{1}, 0.25, 0.0625\}$$

k :	**−2**	**−1**	**0**	**1**	**2**	**3**	**…**	
$x(k)$:			**1**	**1**	1	1	…	
$h(-k)$:	0.0625	0.25	1					$y(0) = 1 \times 1 = 1$
$h(1-k)$		0.0625	0.25	1				$y(1) = 1 \times 0.25 + 1 \times 1 = 1.25$
$h(2-k)$			0.0625	0.25	1			$y(2) = 1 \times 0.0625 + 1 \times 0.25 + 1 \times 1 = 1.3125$
								Stop as required

Boundary Effects

- Quite often nothing is known about input activity that precedes and follows the selection of input samples used for a convolution.
- This means that the calculations of the first few and the last few output samples will be uncertain, because they rely on unknown data.
- These output samples are said to be influenced by boundary effects.
- In analyzing an output signal, it is usually best to discount these samples.
- Fortunately, real signal analyses generally involve thousands of samples, so neglecting a few at the beginning and end will not have a major impact on the output.

Boundary Effects

Example-A of boundary effect which happens when the input sequence x[n] and the impulse response h[n] of the system are not completely overlapped.

Boundary effect can diminish if the impulse response samples are small.

? **= output samples affected by boundary effect.**

x[n]:		1	−2	3	1	5	2	0	1	2	4		
h[-n]:	−1	4											?y[0] = 4
h[1-n]:	2	−1	4										?y[1] = −9
h[2-n]:	−3	2	−1	4									?y[2] = 16
h[3-n]:		−3	2	−1	4								y[3] = −6
…													…
h[9-n]:								−3	2	−1	4		y[9] = 16
h[10-n]:									−3	2	−1	4	?y[10] = −3
h[11-n]:										−3	2	−1	?y[11] = 2
h[12-n]:											−3	2	?y[12] = −12

Boundary Effects

• Initial boundary effects may also be interpreted as output transients.

• **Transient** behavior is the relatively short-term behavior exhibited by a system output.

• **Steady state** part of the output is the long term behavior.

• FIR (finite impulse response) filters reach a clear steady state because their impulse responses have a finite number of samples, and can therefore can be shifted such that the impulse response samples are completely contained by the input signal, and do not extend into regions of unknown inputs.

• IIR (infinite impulse response) filters never reach a true steady state, because some of the infinite number of impulse response samples must inevitably lie outside the range of known input samples.

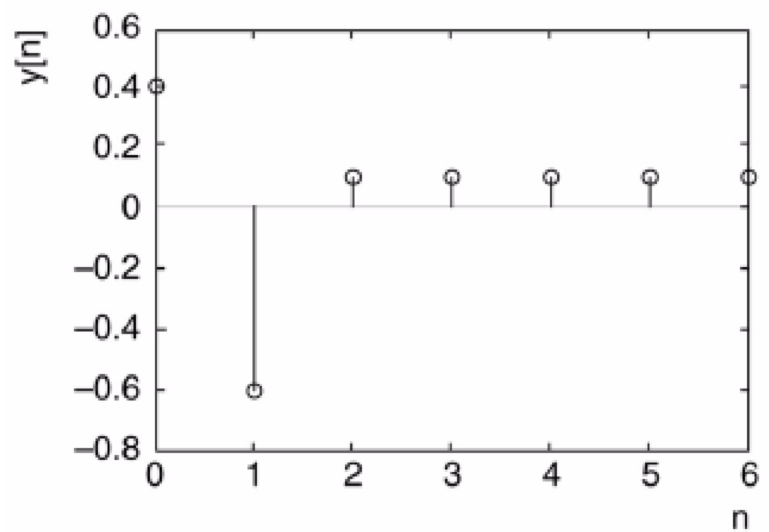
• However, the impulse response samples for stable filters, the only kind normally used, grow smaller with time.

• Thus, an approximate steady state is reached when only very small impulse response samples are combined with unknown inputs.

Boundary Effects

Example-B: The input to a system is the unit step u[n]. The impulse response of the system is given by h[n] = 0.4δ[n] – δ[n - 1] +0.7δ[n - 2]. Find the output of the system using convolution and identify the transient and steady state portion of the output.

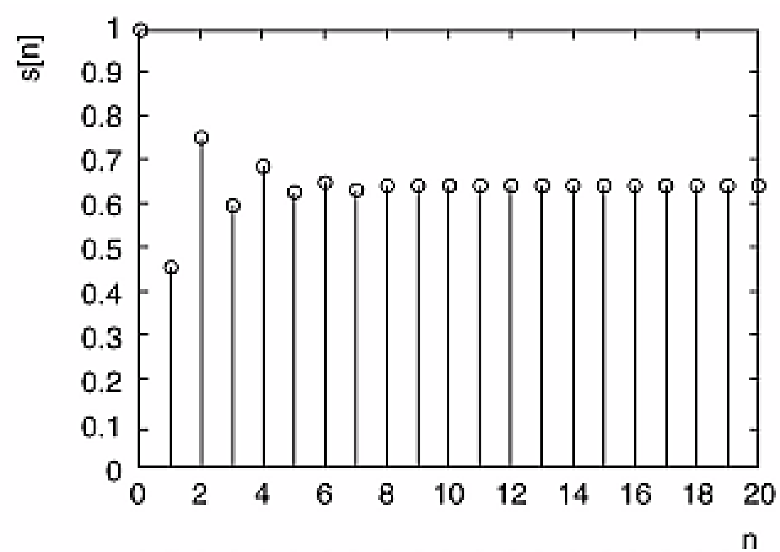
$x[k]$:			1	1	1	1	1	1	1	
$h[-k]$:	0.7	−1	0.4							$y[0] = 0.4$
$h[1-k]$:		0.7	−1	0.4						$y[1] = -0.6$
$h[2-k]$:			0.7	−1	0.4					$y[2] = 0.1$
$h[3-k]$:				0.7	−1	0.4				$y[3] = 0.1$
$h[4-k]$:					0.7	−1	0.4			$y[4] = 0.1$
$h[5-k]$:						0.7	−1	0.4		$y[5] = 0.1$
$h[6-k]$:							0.7	−1	0.4	$y[6] = 0.1$



Boundary Effects

Example-C: Use convolution to find the step response of the system whose impulse response is $h[n] = (-0.55)^n u[n]$

$x[k]$:			1	1	1	1	1	1	1	
$h[-k]$:	0.30	−0.55	1.00							$y[0] = 1.0$
$h[1-k]$:	−0.17	0.30	−0.55	1.00						$y[1] = 0.45$
$h[2-k]$:	0.09	−0.17	0.30	−0.55	1.00					$y[2] = 0.75$
$h[3-k]$:	−0.05	0.09	−0.17	0.30	−0.55	1.00				$y[3] = 0.59$
$h[4-k]$:	0.03	−0.05	0.09	−0.17	0.30	−0.55	1.00			$y[4] = 0.68$
$h[5-k]$:	−0.02	0.03	−0.05	0.09	−0.17	0.30	−0.55	1.00		$y[5] = 0.63$
$h[6-k]$:	0.01	−0.02	0.03	−0.05	0.09	−0.17	0.30	−0.55	1.00	$y[6] = 0.66$

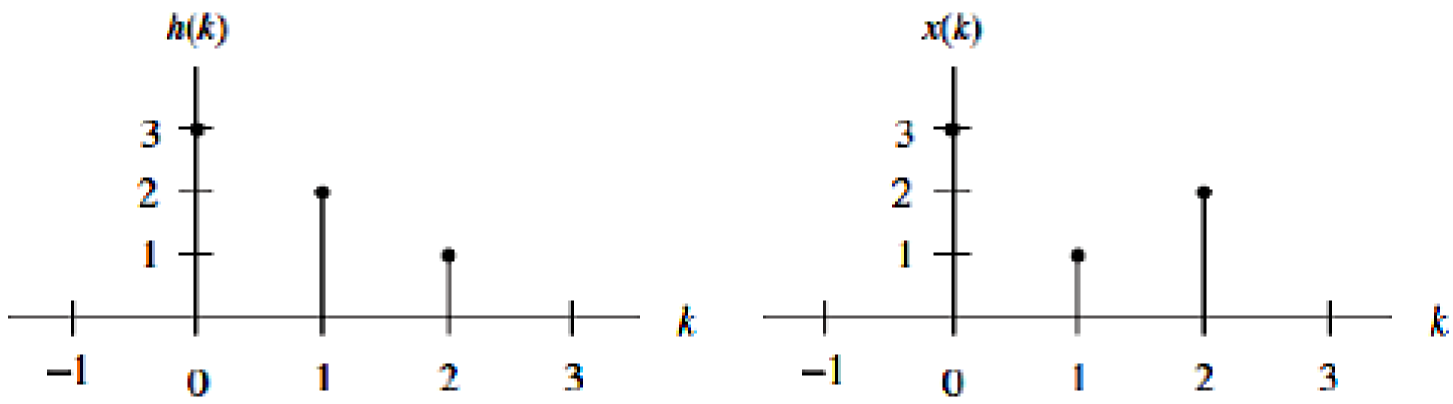


Graphical Digital Convolution

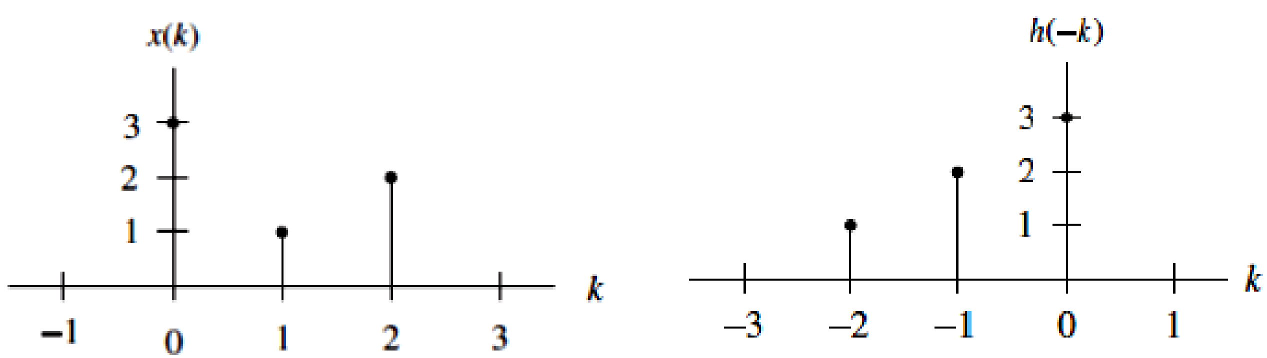
- Step 1. Obtain the reversed sequence $h(-k)$.
- Step 2. Shift $h(-k)$ by $|n|$ samples to get $h(n-k)$.
  - If $n \geq 0$, $h(-k)$ will be shifted to right by n samples.
  - If $n < 0$, $h(-k)$ will be shifted to the left by $|n|$ samples.
- Step 3. Perform the convolution sum, which is the sum of the products of two sequences $x(k)$ and $h(n-k)$, to get $y(n)$.
- Step 4. Repeat Steps 1 to 3 for the next convolution value $y(n)$.

Graphical Digital Convolution

Example-5: Using the sequences defined in Figure, evaluate the digital convolution.



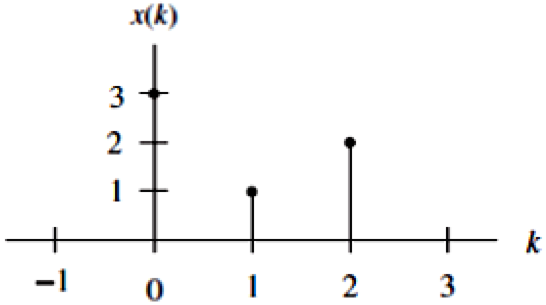
- To get $y(0)$, we need the reversed sequence $h(-k)$.

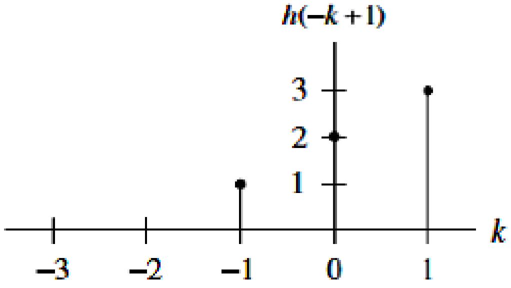


- sum of product of $x(k)$ and $h(-k)$: $\mathbf{y(0) = 3 \times 3 = 9}$

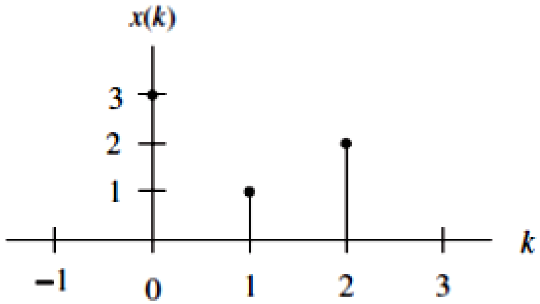
Graphical Digital Convolution

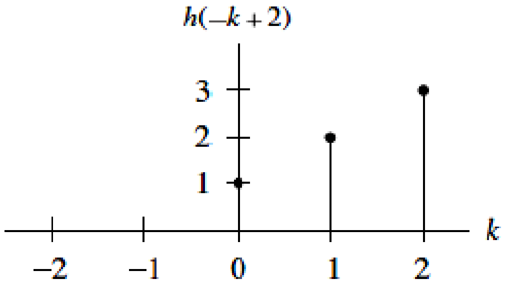
- To get $y(1)$, we need the reversed sequence $h(-k+1)$.





- sum of product of $x(k)$ and $h(1-k)$: $\mathbf{y(1) = 3 \times 2 + 1 \times 3 = 9}$
- To get $y(2)$, we need the reversed sequence $h(-k+2)$.

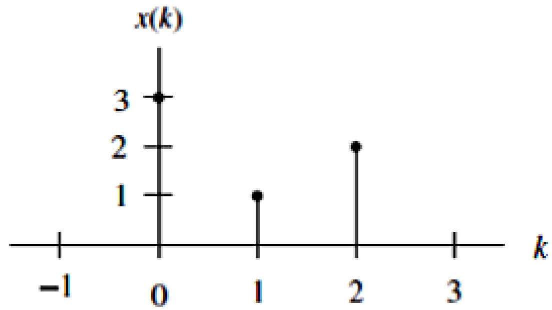


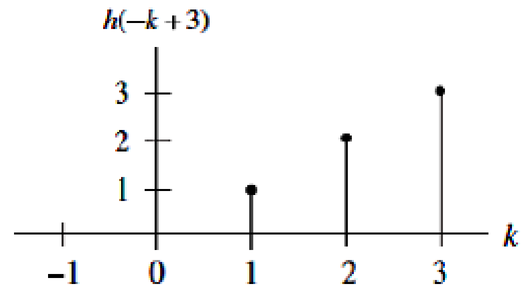


- sum of product of $x(k)$ and $h(2-k)$: $\mathbf{y(2) = 3 \times 1 + 1 \times 2 + 2 \times 3 = 11}$

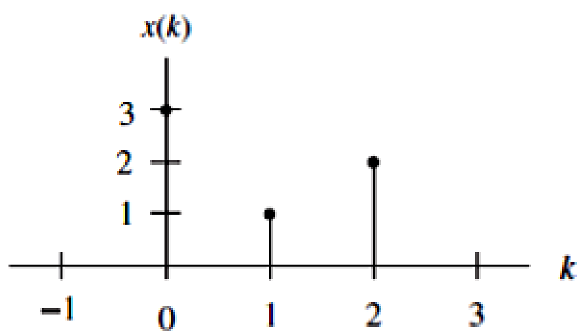
Graphical Digital Convolution

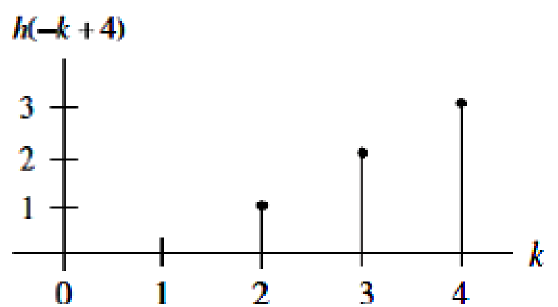
- To get $y(3)$, we need the reversed sequence $h(-k+3)$.





- sum of product of $x(k)$ and $h(3-k)$: $\mathbf{y(3) = 1 \times 1 + 2 \times 2 = 5}$
- To get $y(4)$, we need the reversed sequence $h(-k+3)$.

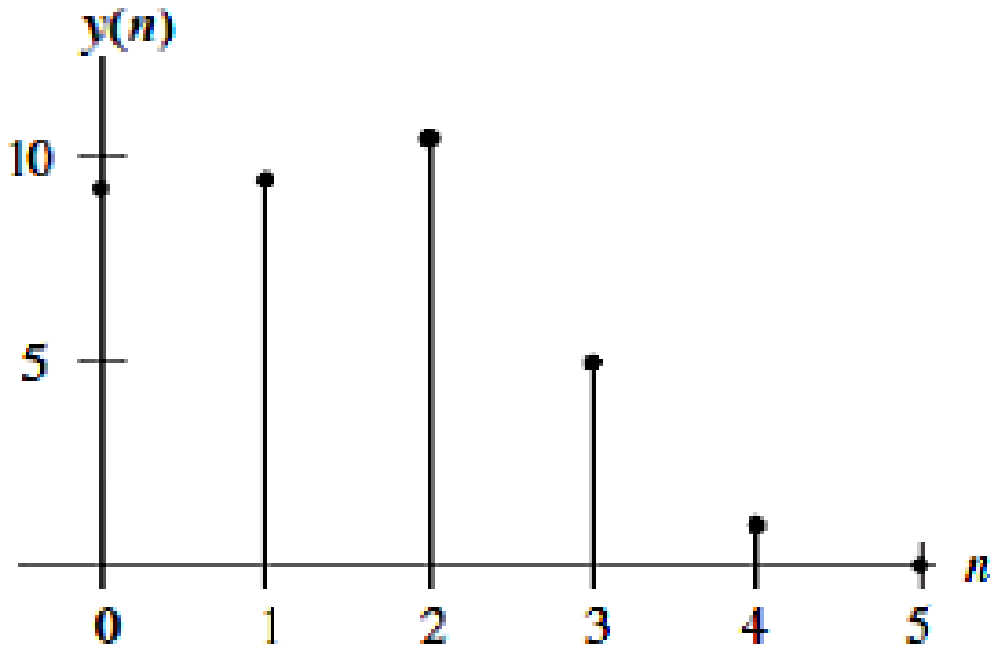




- sum of product of $x(k)$ and $h(4-k)$: $\mathbf{y(4) = 2 \times 1 = 2}$

Graphical Digital Convolution

- sum of product of $x(k)$ and $h(5-k)$: $y(5) = 0$
- For $n > 4$, since sequences $x(k)$ and $h(n-k)$ **do not overlap.**
- Finally, we sketch the output sequence $y(n)$ in Figure



Y[n] = [9, 9, 11, 5, 2]

Graphic Digital Convolution

Example-6:

Input Signal

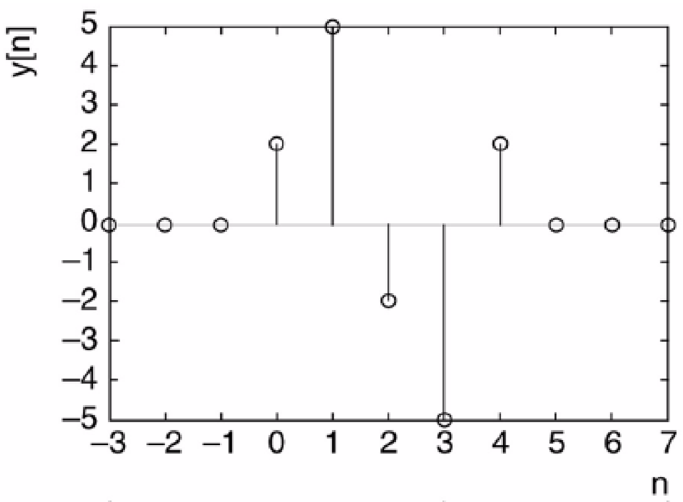
x[n] = [2, 1, 2]

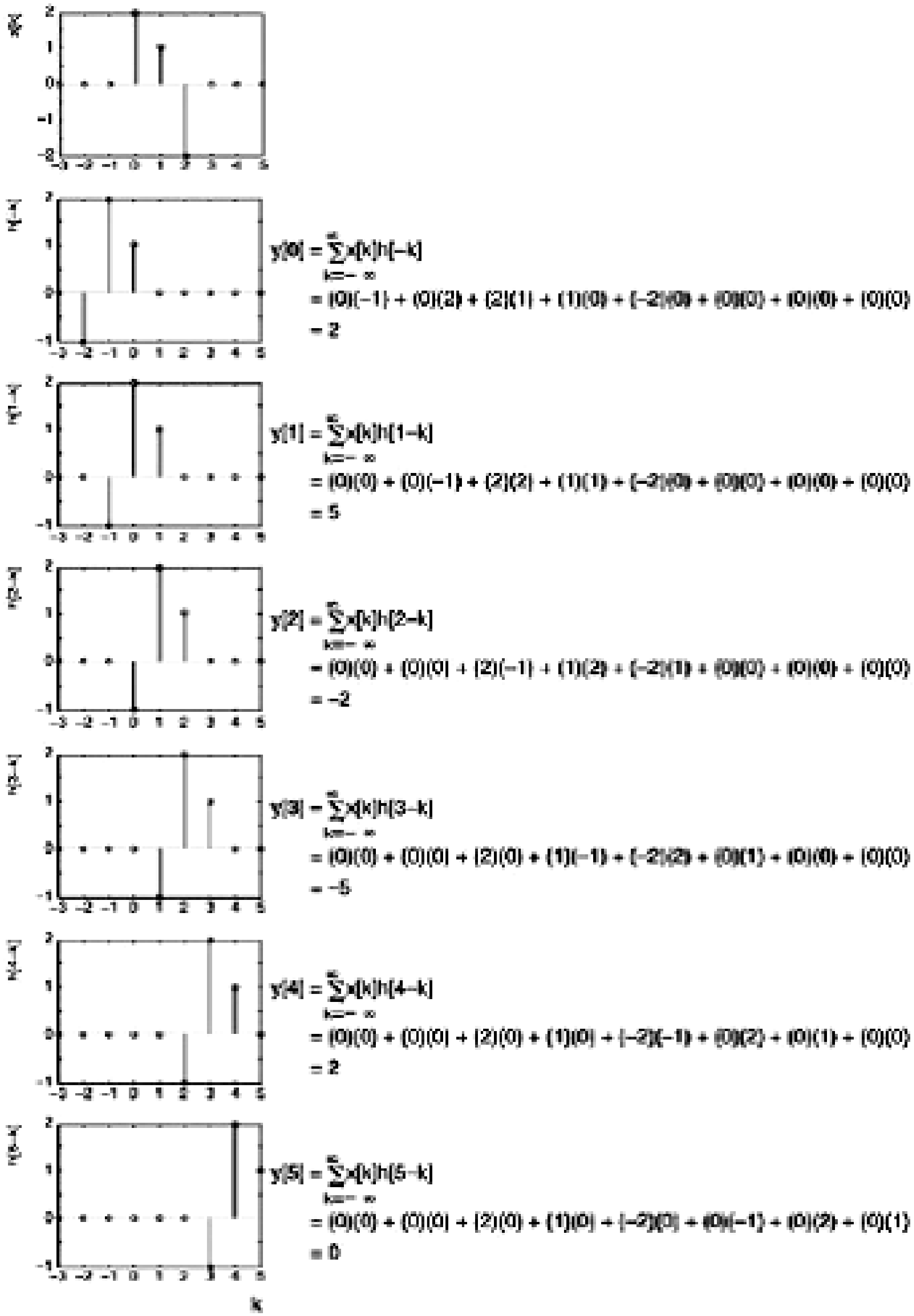
Impulse Response

h[n] = [1, 2, -1]

Output Signal

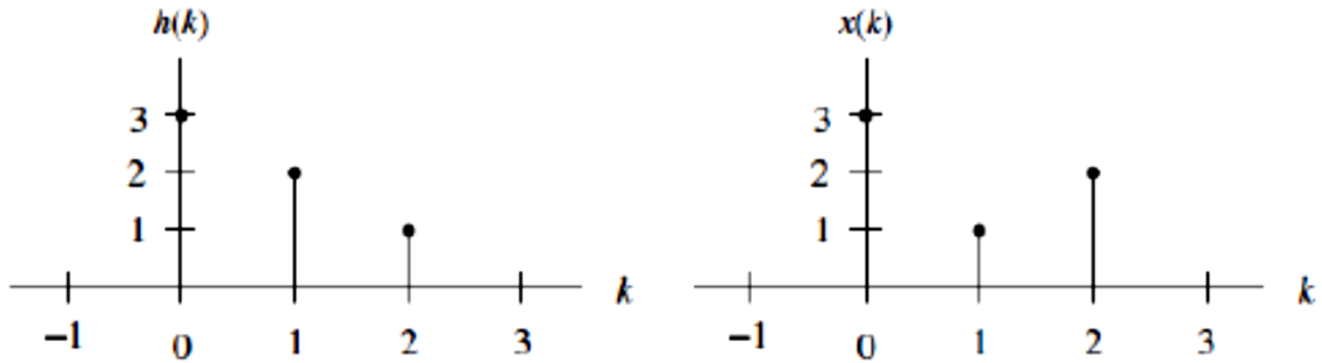
Y[n] = [2, 5, -2, -5, 2]





Convolution by Formula Method

Example-7: Using the sequences defined in Figure, evaluate the digital convolution.



$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$$y(n) = x(0)h(n) + x(1)h(n-1) \;\; + x(2)h(n-2)$$

- $n = 0, y(0) = x(0)h(0) + x(1)h(-1) \;\; + x(2)h(-2) = 9$
- $n = 1, y(1) = x(0)h(1) + x(1)h(0) \;\; + x(2)h(-1) = 9$
- $n = 2, y(2) = x(0)h(2) + x(1)h(1) \;\; + x(2)h(0) = 11$
- $n = 3, y(3) = x(0)h(3) + x(1)h(2) \;\; + x(2)h(1) = 5$
- $n = 4, y(4) = x(0)h(4) + x(1)h(3) \;\; + x(2)h(2) = 2$
- $n \geq 5, y(n) = 0$

Y[n] = [9, 9, 11, 5, 2]

Properties of Convolution

Commutative…

$$x_1[n]*x_2[n]=x_2[n]*x_1[n]$$

Associative…

$$\{x_1[n]*x_2[n]\}*x_3[n]=x_1[n]*\{x_2[n]*x_3[n]\}$$

Distributive…

$$\{x_1[n]+x_2[n]\}*x_3[n]=x_1[n]*x_3[n]+x_2[n]*x_3[n]$$

Correlation

- A measure of similarity between a pair of energy signals, x[n] and y[n], is given by the cross-correlation sequence $r_{xy}[l]$

- Where the parameter *l* is called lag, indicating the time-shift between the pair of signals.

Correlation

- There are applications where it is necessary to compare one reference signal with one or more signals to determine the similarity between the pair and to determine additional information based on the similarity.

- In digital communications, a set of data symbols are represented by a set of unique discrete-time sequences.

- If one of these sequences has been transmitted, the receiver has to determine which particular sequence has been received, by comparing the received signal with every member of possible sequences from the set.

- Similarly correlation can also be used for timing or distance recovery purpose (e.g., RADAR, SONAR, CDMA receiver, Ultrasound etc.)

Cross Correlation

- For two infinite sequences

$$\Gamma_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)y(n-l)$$

- Equivalently:

$$\Gamma_{yx}(l) = \sum_{n=-\infty}^{\infty} y(n)x(n-l)$$

$$\boxed{\Gamma_{xy}(l) = \Gamma_{yx}(-l)}$$

Cross Correlation

The number of samples *N* in the output signal will be

$$N = M_1 + M_2 - 1$$

Where
M_1 is the number of samples in sequence $x_1[x]$
M_2 is the number of samples in sequence $x_2[x]$

Cross Correlation

Example-8: Find the correlation b/w the two sequences x[n] and y[n] given by,

$$x[k] = [\underset{\uparrow}{3}\ 1\ 2] \qquad y[k] = [3\ \underset{\uparrow}{2}\ 1]$$

k:	-2	-1	0	1	2	3	4	5
x[k]:			3	1	2			
y[k+2]:	3	2	1					
y[k+1]:		3	2	1				
y[k]:			3	2	1			
y[k-1]:				3	2	1		
y[k-2]:					3	2	1	

$r_{xy}[-2] = 3 \times 1 = 3$

$r_{xy}[-1] = 3 \times 2 + 1 \times 1 = 7$

$r_{xy}[0] = 3 \times 3 + 1 \times 2 + 2 \times 1 = 13$

$r_{xy}[1] = 1 \times 3 + 2 \times 2 = 7$

$r_{xy}[2] = 2 \times 3 = 6$

$$\boxed{\Gamma_{xy}(l) = \{3 \quad 7 \quad 13 \quad 7 \quad 6\}}$$

Cross Correlation

Example-9: Find the correlation b/w the two sequences x[n] and y[n] given by,

$$x[k] = [\underset{\uparrow}{3}\ 1\ 2] \qquad y[k] = [\underset{\uparrow}{3}\ 2\ 1]$$

k:	-2	-1	0	1	2	3	4	5
y[k]:			3	2	1			
x[k+2]:	3	1	2					
x[k+1]:		3	1	2				
x[k]:			3	1	2			
x[k-1]:				3	1	2		
x[k-2]:					3	1	2	

$r_{yx}[-2] = 3 \times 2 = 6$

$r_{yx}[-1] = 3 \times 1 + 2 \times 2 = 7$

$r_{yx}[0] = 3 \times 3 + 2 \times 1 + 1 \times 2 = 13$

$r_{yx}[1] = 2 \times 3 + 1 \times 1 = 7$

$r_{yx}[2] = 1 \times 3 = 3$

$$\boxed{\Gamma_{yx}(l) = \{6 \quad 7 \quad 13 \quad 7 \quad 3\}}$$

Cross Correlation

$$\boxed{\Gamma_{xy}(l) = \{3 \quad 7 \quad 13 \quad 7 \quad 6\}} \qquad \boxed{\Gamma_{yx}(l) = \{6 \quad 7 \quad 13 \quad 7 \quad 3\}}$$

- Comparing the two establishes that

$$\Gamma_{xy}(l) = \Gamma_{yx}(-l)$$

Cross correlation does not exhibit Commutative

$$\Gamma xy(l) \neq \Gamma yx(l)$$

Cross Correlation

Example-10: Find the correlation of the two sequences x[n] and y[n] represented by,

x[n] = [1, 2, 3, 4] **y[n] = [5, 6, 7, 8]**

Solution

$$Y_{xy}[n] = [8, 23, 44, 70, 56, 39, 20]$$

Cross Correlation

Example-11: Find the correlation of the two sequences x[n] and y[n] represented by,

x[n] = [1, 1, 1, 1] **y[n] = [1, 2, 3]**

Solution

$$Y_{xy}[n] = [3, 5, 6, 6, 3, 1]$$

Cross Correlation

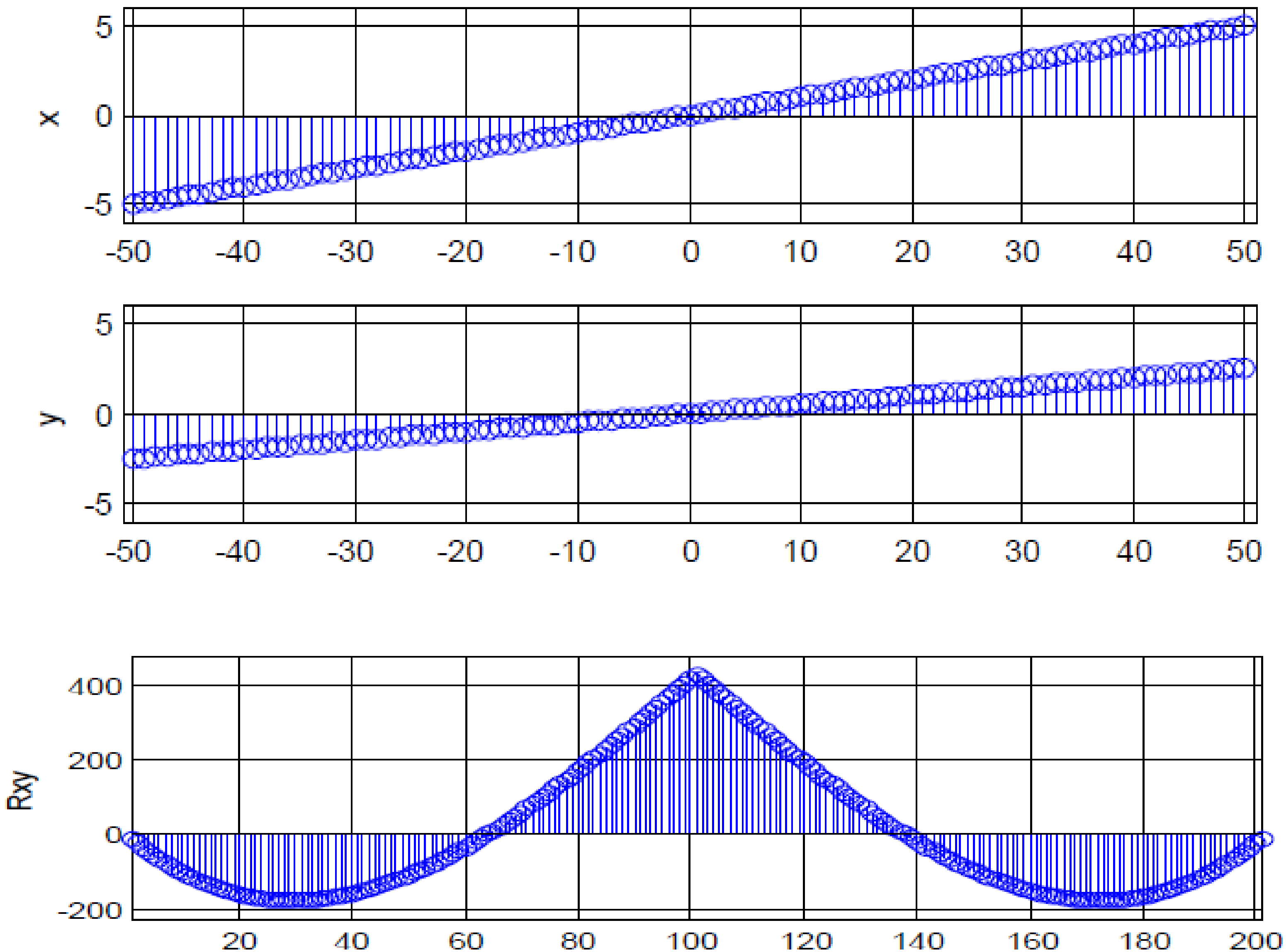
Excersize-1: Find the correlation of the two sequences x[n] and y[n] represented by,

$$x[n] = [\underset{\uparrow}{1}\ \ 2\ \ 4] \qquad y[n] = [\underset{\uparrow}{1}\ \ 1\ \ 1\ \ 1\ \ 1]$$

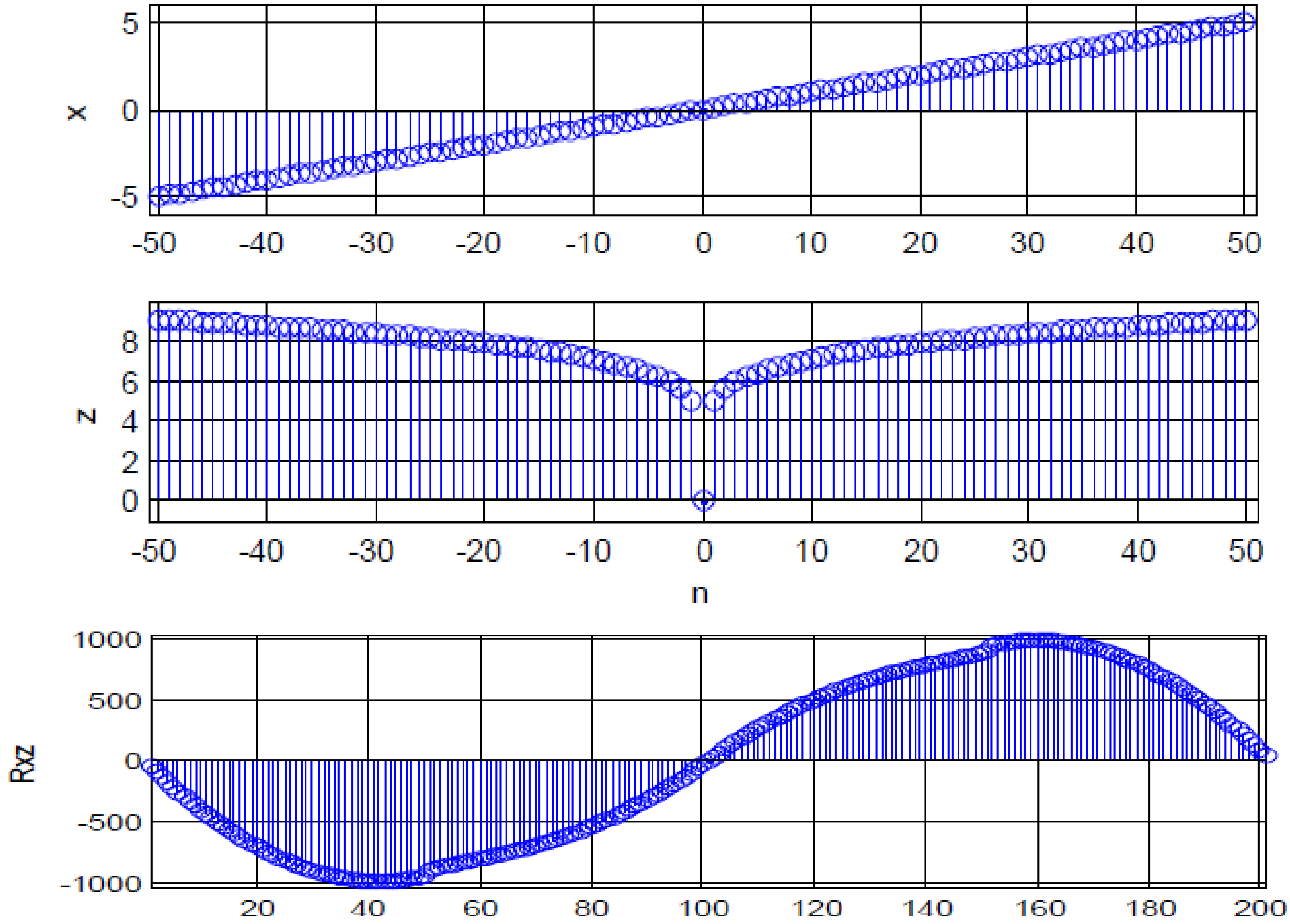
Excersize-2: Find the correlation of the two sequences x[n] and y[n] represented by,

$$x[n] = \{\underset{\uparrow}{0}\ \ 1\ \ -2\ \ 3\ \ -4\} \qquad y[n] = [0.5\ \ 1\ \ \underset{\uparrow}{2}\ \ 1\ \ 0.5]$$

Correlation Between Signals X and Y



Correlation Between Signals X and Z



Auto Correlation

- Correlation of a function with itself

$$\Gamma_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)y(n-l)$$

- If y(n)=x(n)

$$\Gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$$

- For no shift, i.e. l=0

$$\Gamma_{xx}(0) = \sum_{n=-\infty}^{\infty} x^2(n) = E_x$$

Auto Correlation

- **Recovering a repeating pattern, or any periodic signal from its highly-noisy version**

- **Recovering fundamental frequency of an otherwise random signal**

Auto Correlation

The number of samples ***N*** in the output signal will be

$$N = 2 \times M - 1$$

Where,

M is the number of samples in the sequence x[n]

Auto Correlation

Example: Find the auto correlation of the following sequence

$$x[n] = [1, 2, 3, 4]$$

Solution

$$Y_{xx}[n] = [4, 11, 20, 30, 20, 11, 4]$$

Auto Correlation

Example: Find the auto correlation of the following sequence

$$x[n] = [1, 2, 3, 4]$$

Solution

$$Y_{xx}[n] = [4, 11, 20, 30, 20, 11, 4]$$